Developing Japanese High School Students' Consciousness of the Interrelations between Mathematics and Science

——Through Mathematical Modelling Experiences——

Tetsushi KAWASAKI

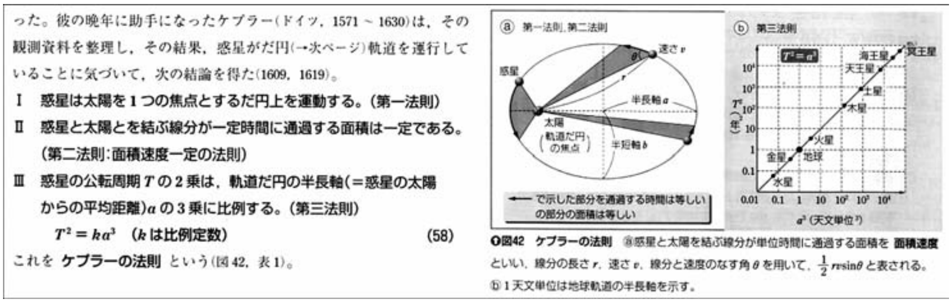
Abstract: *The estrangement of science from mathematics senior high school mathematical education in Japan is the subject of widespread grave concern. Thus, in order to connect mathematics to the fields of Newton's science, for example, through mathematical modeling, it is suggested that it will be necessary to provide concrete examples such as with "Kepler's Laws". This approach succeeded in increasing a class of Year 12 students' knowledge about the laws with the simulation of planetary movements and the making of a "Mathematical development model" occurring together. Consequently students will realize the necessity of differential equations in order to analyze actual phenomena. This empirical research suggests that mathematics materials involving physical perspectives are effective for senior high school students.*

1. Introduction

In school education in Japan, it is thought that it is enough for mathematics teachers to teach only theory. It is believed that the science teachers practice the application and the use of mathematics; but it has not been actually achieved. Mathematics is removed away from the phenomena of daily life and science. Students will understand the ideas through the formal theory of mathematics; but they have not gleaned the scientific spirit. Additionally, it is difficult for them to understand the concept of mathematics theory.

The reason why students study mathematics is "to notice its necessity and to learn the thought". The solution of problems by mathematical modeling is necessary to achieve these two purposes.

About the mathematics that aimed at the science, Fujii, J. (1986) pointed out the following problems; "The calculus education in the Japanese senior high school is the unit to be able to learn a theory of the Newton's science. However, the senior high school mathematics does not aim at the dynamics. An educational practice to lead the elliptic orbit of the planet used Newton's law of gravitation is necessary". The differential equation by the law of universal gravitation (Newton's Law) is necessary to prove Ke-


った。彼の晩年に助手になったケプラー(ドイツ，1571～1630)は，その観測資料を整理し，その結果，惑星がだ円(→次ページ)軌道を運行していることに気づいて，次の結論を得た(1609，1619)。

Ⅰ　惑星は太陽を１つの焦点とするだ円上を運動する。(第一法則)

Ⅱ　惑星と太陽とを結ぶ線分が一定時間に通過する面積は一定である。(第二法則：面積速度一定の法則)

Ⅲ　惑星の公転周期 T の２乗は，軌道だ円の半長軸(＝惑星の太陽からの平均距離)a の３乗に比例する。(第三法則)

$$T^2 = ka^3 \quad (k\text{ は比例定数}) \qquad (58)$$

これを **ケプラーの法則** という(図42，表1)。

図42　ケプラーの法則　ⓐ惑星と太陽を結ぶ線分が単位時間に通過する面積を **面積速度** といい，線分の長さ r，速さ v，線分と速度のなす角 θ を用いて，$\frac{1}{2}rv\sin\theta$ と表される。
ⓑ１天文単位は地球軌道の半長軸を示す。

Fig. 1　Formal knowledge and the illustration of Kepler's Laws
(An excerpt from our high school physics textbook)

pler's Laws. However, high school physics doesn't prove Kepler's Laws. The illustration of the physics textbook (Year 11) is only presented (Fig. 1). High school mathematics mainly treats the ellipse in orthogonal axes coordinates. High school physics mainly treats uniform circular motion. Both high school mathematics and physics only show formal knowledge. Students will not be able to notice the necessity of mathematics or be able to learn correct thought by applying mathematics. The result is uncertain that they can gain the spirit of modern science well by the situation of the school education. Kawano, Y. (2001) was a class by this mathematical modeling. There were not the data of the students; recognition change and understanding and satisfaction.

We carried out the following research (Fig. 2). It shows the low knowledge of the students in a Year 12 class (N＝39), where high school attached to Kyoto University of education, to the mathematics background theory used for Kepler's Laws. They do not know the deep meaning though they know the formal theory. The students know that a planet goes around the sun. They learned the Kepler's law in Year 11. However, they learned eccentricity by the mathematics class, but they do not remember even words and the meaning either. In addition, they have not had the class that they used a computer for. Perhaps it is thought that the students would not understand "the common point of the focus and the

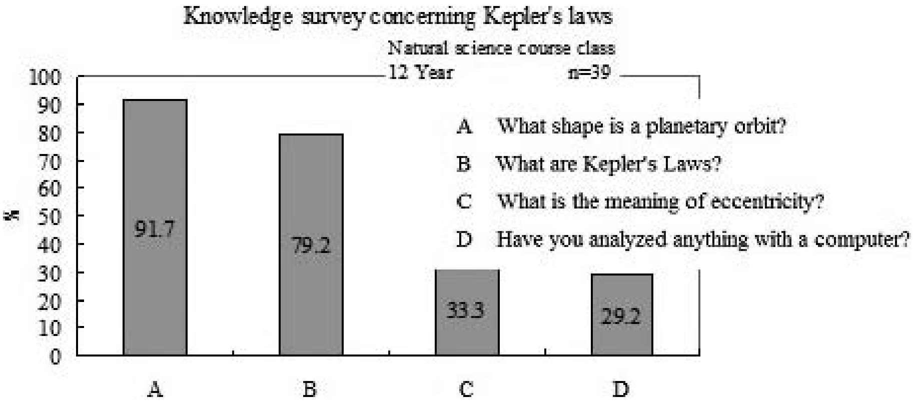


Fig. 2　Knowledge survey concerning Kepler's Laws

sun" and "the planet's motion".

When we analyze a real event that needs mathematics, it is necessary for us to apply mathematics (Applied Problem) and it is necessary for us to think about the solution by understanding the meaning of the problem. We practice this, and it becomes necessary to look back at the results as in the modelling cycle[1)] process Fig. 3 (a)[2)].

Then, when the actual model describes the scene and the case of reality, we call it an " Elementary Model" (see Fig. 3 (b)). The model described by having mathematics that the students studied is called a "Pre-model", because it becomes a part of, and a connection to, the development model though it is dissatisfactory as a model. The model which guides students to the new mathematics is called a "Mathematical development model". These models provide the basis for introductory teaching materials of the new mathematics unit (Fig. 3 (b) where students develop an image of modelling).

When the model changes from simplicity into complexity, students repeat this modeling. However, it is advisable that the student do not think about this process alone, because they can't find the new model from only their experiences. A minimum of support by the teachers is necessary (cf. Blum, W., & Leiß, D. (2007)).

An example of this modeling is the teaching materials using "Kepler's Law" and the law of universal gravitation which will be explained in the next section.

To make high school students consider how mathematics is connected with science is very difficult. Why is it difficult?

The following educational situations have always existed in Japan. According to the course of study (2005)[3)] in Japan, all teachers in Japan must hew to curriculum guidelines that commonwealth institution managed. Teachers should use the textbooks that stick to its rules. The guidelines had not treated the curriculum of mathematical modeling before now. So in mathematics and science education in Japan, the curriculum for modeling activities doesn't exist. The school system in Japan doesn't permit mathematics teachers to teach other subjects such as physics or information processing. Also educational materials beyond the guidelines aren't on the test in the university examination. Teachers don't have cir-

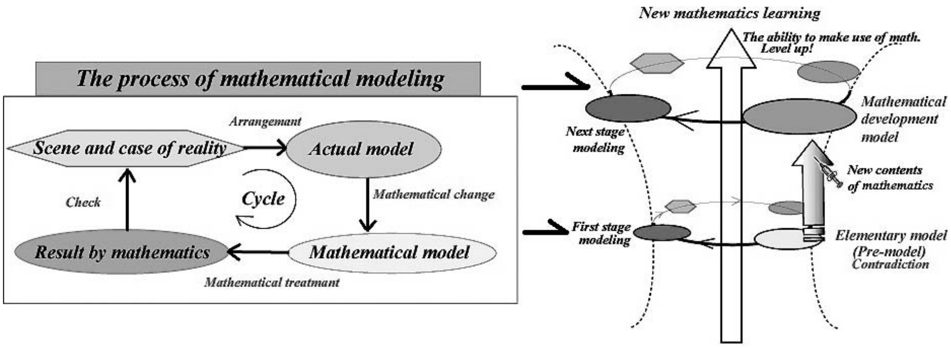


Fig. 3 (a) Mathematical modeling (cf. Kaiser, 1995) (b) The development of modeling image

cumstance that they develop teaching materials so much. So teachers will depend on the textbook. In addition, their classes emphasize mathematics to get into university by students' guardian's demand. Therefore the reason why students study mathematics became studying for the entrance examination. A lot of high school students don't have basic knowledge of the connections between mathematics and other areas (Fig. 2). Not only the students but also teachers will become to lack the recognition of the necessity of mathematics.

To address such situations, new teaching materials need to be developed.

2. A practice example using this modeling

－Teaching materials using Kepler's Law for high school students becoming scientists－

This practice is not the only suggestion of this modeling. In this practice, there is the meaning that we should solve the problems of school education from the preceding chapter. We want to recommend the proof of the elliptical orbit of the planet as a mathematics material with physical viewpoints, because the essence of modern science is connected to Keplerian and Newtonian science.

2. 1. Pre-model

First of all, Students will practice by using the observational data of Mercury. This will be named the elementary model to understand Kepler's Law.

The elliptical orbit of Mercury appears by the crowd of the tangential lines, "Kepler's 1st Law" (Fig. 5). And, when the length of observational periods are equal, the sectoral areas caused by the segment that connects the sun with Mercury are equal, "Kepler's 2nd Law". Usually only this model must be satisfactory, and as a result, we think that students will confirm the image of Kepler's Law.

However, it is doubtful whether the students understand these rightly. In fact we showed them the simulation of two planetary movements (Fig. 6), and we made them judge which was correct. At once

Table 1　The observational data of Mercury

Year	Eastern Max angle (E)		Western Max angle (W)	
1990	April 14	20″	February 1	25″
	August 12	27″	May 31	25″
	December 6	21″	September 24	18″
1991	March 27	27″	January 14	24″
	July 25	27″	May 13	26″
	November 19	22″	September 8	18″
			December 28	22″
1992	March 10	18″	April 23	27″
	July 6	26″	August 21	19″
	November 1	24″	December 9	21″

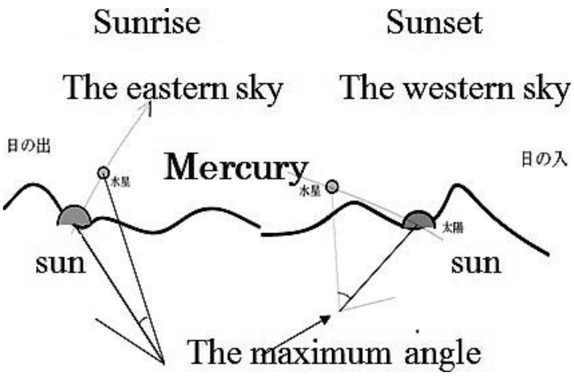


Fig. 4　The maximum angle

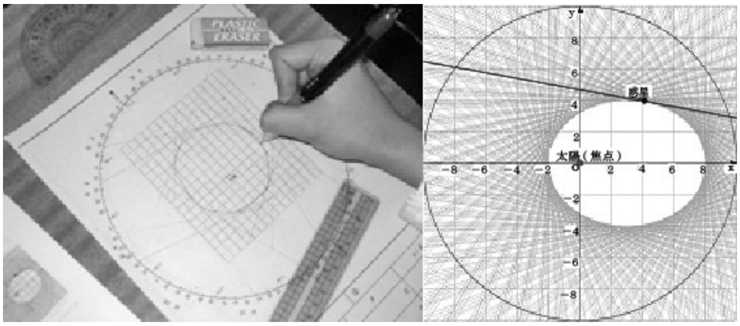


Fig. 5 Drawing in Mercury orbit by a lot of tangential lines
(right) elementary model, (left) pre-model

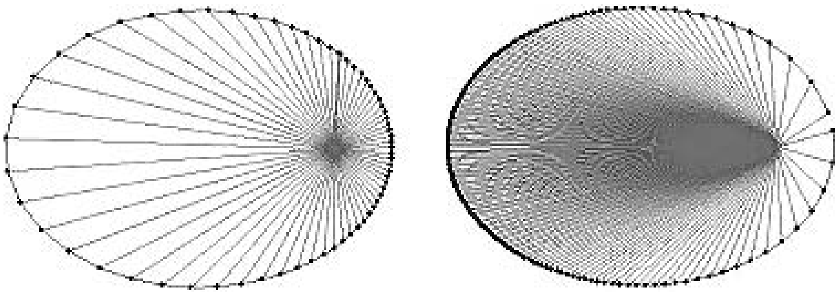


Fig. 6 (a) Uniform motion (b) Planetary motion

students couldn't judge, and selected Fig. 6 (a) later. The correct answer is Fig. 6 (b).

This elementary model is not enough for students' knowledge to mature. But it has some good result as a model. Before long they will demand a new model. Therefore, we call it the pre-model that prepare for a new model. It is necessary to prepare the new contents of mathematics. The new model is shown in the next section. It becomes one composition by these two models. The new model has a means to lead to a new mathematics unit. Students will make the best use of the new model for the next practice.

2. 2. Mathematical development model

We newly took the differential equation. It is necessary to prepare new contents to apply mathematics, and it becomes an introduction of the differential equation. The equation of Kepler's Laws by Newton's Law is a second order linear differential equation with a scalar type constant number. This is very difficult and they can't find the new model from only their experiences.

A minimum of support by the teachers is necessary, and I tried to help the students' gain understanding, using the analysis method of numerical values by computer programming. They will be able to have full realization of Kepler's Laws and Newton's Law by drawing the solution curve using Euler's method (Fig. 7).

The solution curve by Euler's method has the fault that the error grows when the input value parts

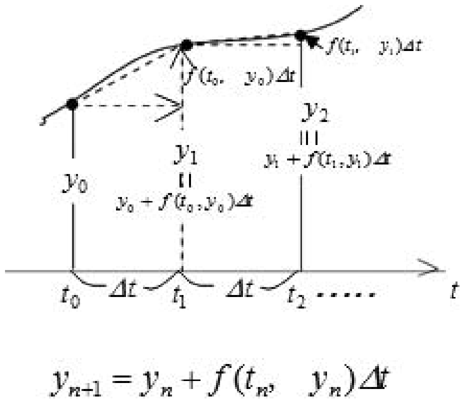


Fig. 7 Euler method

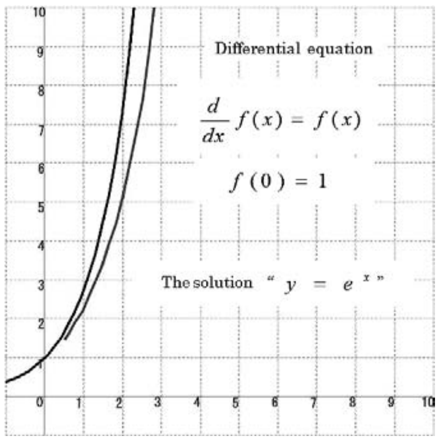


Fig. 8 By Euler method

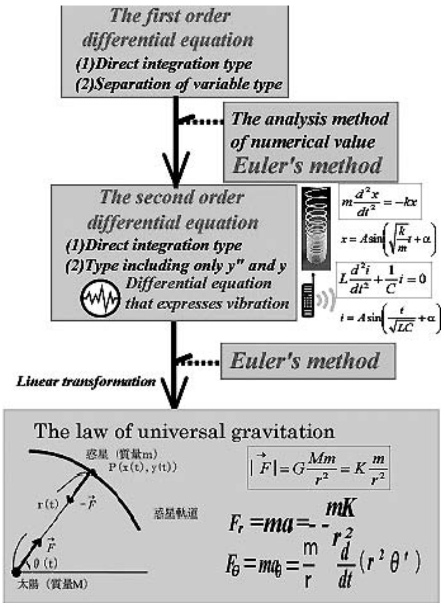


Fig. 9 System diagram to introduce differential equations

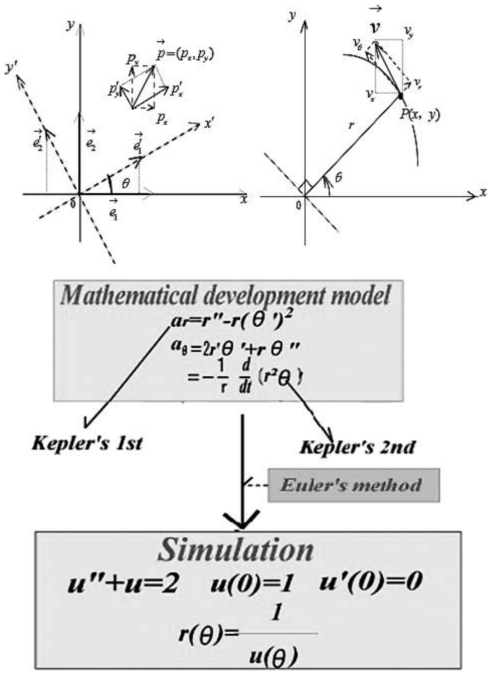


Fig. 10 Transform of coordinates and Mathematical development model

from an initial value (Fig. 8). When teachers give the student the base program, regard to the error is necessary. As for this feature, the simple harmonic motion in the high school physics can be treated by the differential equation. And this second order differential equation relates to the law of universal gravitation. I will show the system diagram to introduce differential equations (Fig. 9), and Fig. 10 is the mathematical development model with transform of coodinates.

Contents of other necessary mathematics are polar coordinates and linear transformations. Students could smoothly make the mathematical development model by the work-sheet and computer program-

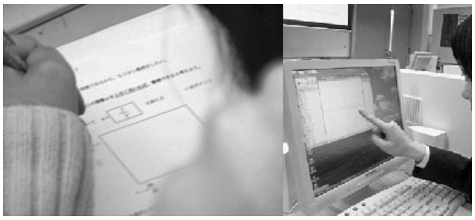


Fig. 11　The work-sheet and computer programming

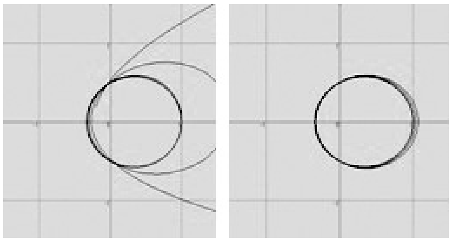


Fig. 12　Simulation result
(left) $d\theta = 0.1$, (right) $d\theta = 0.01$

ming (Fig. 11).

In addition, this mathematical development model means the polar equation of ellipse is,

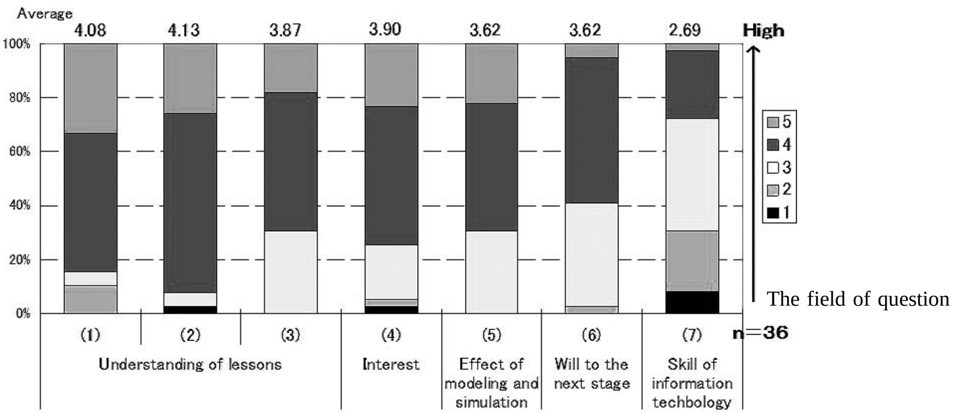
$r = \dfrac{l}{1 - e\cos\theta}$　($l > 0$, $e > 0$), (r: Distance between Sun and planet).

Students confirmed whether this solution accorded with the drawing of the solution curve (Fig. 12).

3. Students' evaluation and impressions

3. 1. The class treated the pre-model

The students understood mathematics in the background of Kepler's Law well. Understanding and interest show high points Fig. 13. It seemed they felt admiration for the work of the planetary orbit. They noticed that Kepler's 2nd law used mathematics they had not learned. However, they couldn't interpret the new mathematics very well. The reason is because they had trouble with choosing the correct an-



(1) "Kepler's 1st Law" by the drawing figures (Elliptic orbit and Envelope)
(2) Mathematics (the background of "Kepler's 1st Law")
(3) Mathematics (the background of "Kepler's 2nd Law")
(4) The mathematics use for the planet orbit
(5) Two planetary movements
(6) New mathematics is necessary
(7) The use of Texas Instrument

Fig. 13　"Pre-model" －Students' evaluation－

swer from two planetary movements. The high school mathematics in Japan (cf. the course of study, 2005) treats only one parameter, and treats function of one variable. In addition, it doesn't treat the state of a momentary movement. The high school physics in Japan displays only the illustration by Kepler's laws, and doesn't proof it. So it seemed that it is difficult for students to image planetary motion, but many of them had will to learn new mathematics (Fig. 13 (6)). There were some students that got a little low points for Fig. 13 (6) and (7) 's evaluations. The reason is because some students are not accustomed to the calculator and also classes advanced fast. The reason is because some students were not accustomed to the calculator, and also the classes advanced fast.

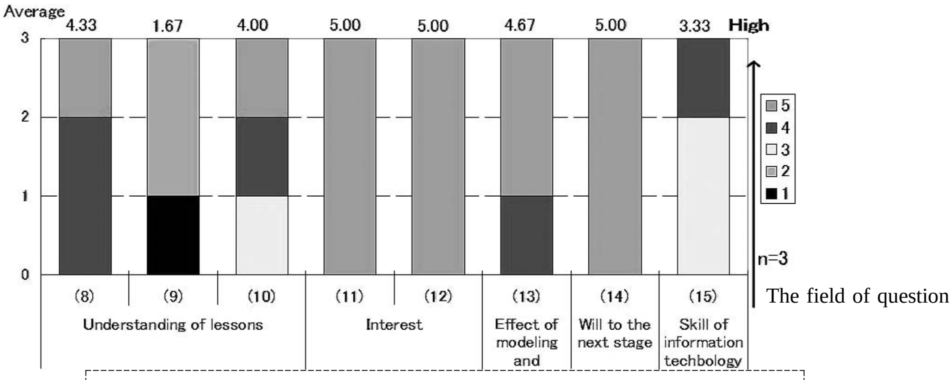
3. 2. The class treated the mathematical development model

At this time, students were busy for the university examination, so this class were carried out for three students.

They entered the science faculty of Kyoto University. They almost appeared very high score.

One student's impression was "This lesson was a valuable experience of learning the interest of natural science before I get into college". Analysis by the programming proved to be very effective in the students' understanding (Fig. 14 (11) (12) (14)).

However, there was a lot of content in a few classes. It seemed that students had difficulty understanding the content though they were interested in it. After all, It was difficult for students to make the model by only one person. Moreover, it was also difficult for the teacher to moderate their work (Fig. 14 (9)).



(8) The solution curve by Euler's method
(9) The differential equation in the worksheet was solved alone.
(10) Proof of Kepler's Law by the teacher's support
(11) Analysis by the programming
(12) A planet motion was showed by the PC.
(13) Confirming the result is effective.
(14) The analysis for other phenomena except Kepler
(15) To the programming

Fig. 14 "Mathematical development model" －Students' evaluation－

Another student's opinion, "Both content of modelling and guidance to Kepler's motion is too greedy". A little more time might have been necessary so that the students might master the knowledge. However, this modeling aims at the introduction into a new mathematics unit like differential equations by making a mathematical development model. The students would surely have experienced this step with enough practice.

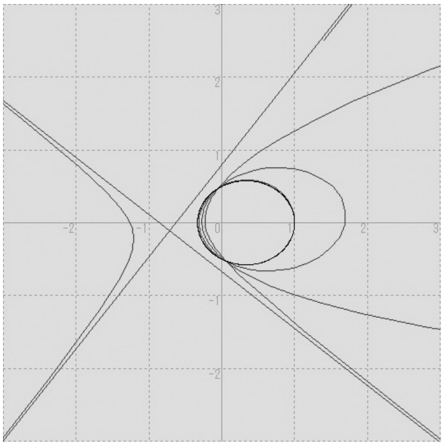


Fig. 15 Exercise that student presented

In the school education of Japan, there is no custom of using information technology effectively. The strong points and weak points of the programming were caused by the students' ability. This is difficult if there is no educational environment that properly treats information technology. Moreover, there was a student who verified the situation in which the eccentricity was changed (Fig. 15).

4. Conclusion and future subjects

Both of two modelings showed utility as an educational content with consciousness of the interrelation between science and mathematics. This practice example of modeling had the result of making students understand the true concept of mathematics theory against the background of a great scientific discoveries. Students have learnt some mathematics knowledge until now. The incomplete model might not go well, when they try to solve the problem applying their mathematics knowledge. It is also important to give students new mathematics knowledge, if the model has contradiction and the limit. The modeling practice in a more advanced stage is also necessary. So teachers should prepare the new contents. Students will surely grow up if the preparation succeeds well. And, this modeling will bridge to a new mathematics unit. When students must make a mathematics model that they have not seen or experienced, teachers should assist whether they need the application of learnt mathematics or whether they need new mathematics.

The high school students and the university students in Japan are not accustomed to problem solving by modeling. There are few chances for them to notice the necessity of mathematics, when students study mathematics. Even if it is at a late time when students treat the modeling, It is necessary to improve the teacher's guidance by applying such modeling. This time, students made the mathematical model with teacher support. It is difficult for teacher to make a judgment about whether it was the minimum of support needed. But in the future, they might have to research in teams, or to solve problems alone. Students had better get as much such self-help as possible at early stages of their growth (cf. Blum, W., & Leiß, D. (2007)).

This practice showed that mathematics was useful for science. Students could catch a glimpse of scientific spirit. Though not all of the students achieved, they experienced and understood the concept of the mathematics theory. Such modeling practice is important for students who aim to become scientists and engineers and for students to become wise citizens.

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